8장 Matrice -6

- 최소좌승법 (Least Square)
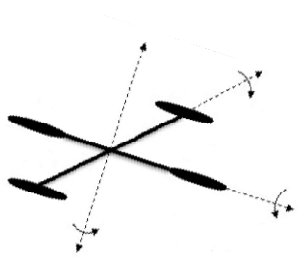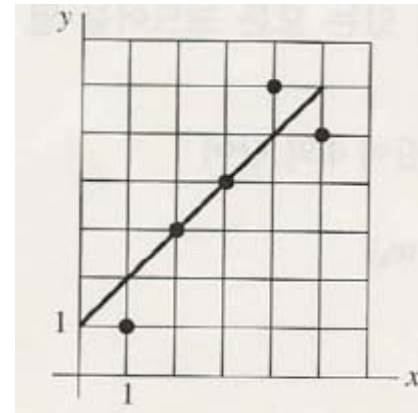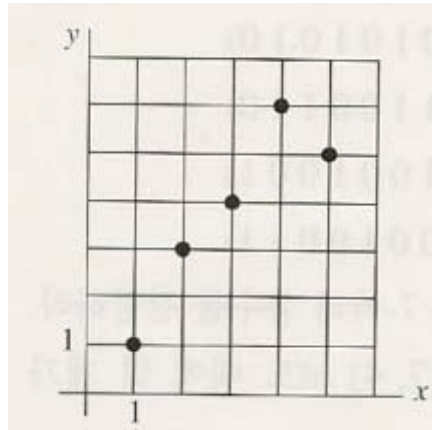
$$y_1 = ax_1 + b$$
$$y_2 = ax_2 + b$$
$$\vdots$$
$$y_n = ax_n + b$$

$$\mathbf{Y} = \mathbf{AX}, \quad \text{여기서} \quad \mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \ \mathbf{A} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix}, \ \mathbf{X} = \begin{pmatrix} a \\ b \end{pmatrix}$$
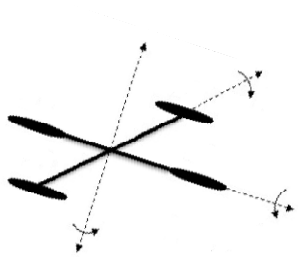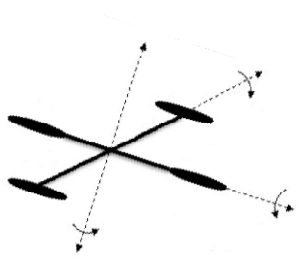
$$e_i = |y_i \hat{} - f(x_i)|, \quad i = 1, 2, \ldots, n \qquad f(x) = ax + b$$

$$E = [y_1 - f(x_1)]^2 + [y_2 - f(x_2)]^2 + \cdots + [y_n - f(x_n)]^2$$
$$= [y_1 - (ax_1 + b)]^2 + [y_2 - (ax_2 + b)]^2 + \cdots + [y_n - (ax_n + b)]^2$$

$$E = \sum_{i=1}^{n} [y_i - ax_i - b]^2$$

$$\frac{\partial E}{\partial a} = 0 \;\; \text{그리고} \;\; \frac{\partial E}{\partial b} = 0$$

$$-2\sum_{i=1}^{n} x_i[y_i - ax_i - b] = 0$$
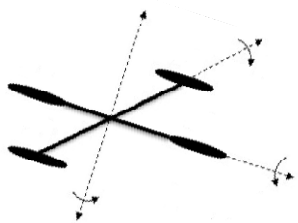
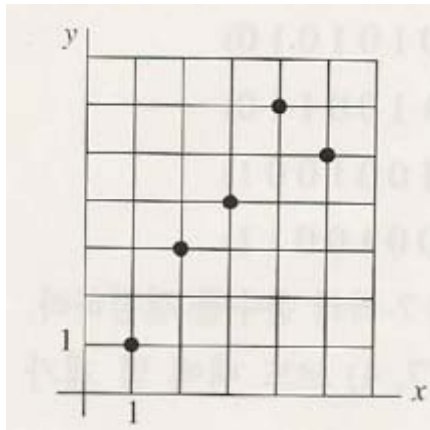$$-2\sum_{i=1}^{n} [y_i - ax_i - b] = 0$$

$\longrightarrow$

$$\left(\sum_{i=1}^{n} x_i^2\right)a + \left(\sum_{i=1}^{n} x_i\right)b = \sum_{i=1}^{n} x_i y_i$$

$$\left(\sum_{i=1}^{n} x_i\right)a + \quad nb = \sum_{i=1}^{n} y_i$$

$$\mathbf{A}^T\mathbf{A}\mathbf{X} = \mathbf{A}^T\mathbf{Y}$$

$$\boxed{\mathbf{X} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}}$$

$(1, 1), (2, 3), (3, 4), (4, 6), (5, 5)$
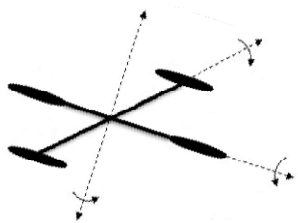
$$a + b = 1$$
$$2a + b = 3$$
$$3a + b = 4$$
$$4a + b = 6$$
$$5a + b = 5$$

$$\mathbf{Y} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 6 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{pmatrix} \quad \text{we have} \quad \mathbf{A}^T\mathbf{A} = \begin{pmatrix} 55 & 15 \\ 15 & 5 \end{pmatrix}$$

$$\mathbf{X} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}$$

$$\mathbf{X} = \begin{pmatrix} 55 & 15 \\ 15 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ 3 \\ 4 \\ 6 \\ 5 \end{pmatrix} = \frac{1}{50} \begin{pmatrix} 5 & -15 \\ -15 & 55 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \\ 6 \\ 5 \end{pmatrix}$$

$$= \frac{1}{50} \begin{pmatrix} 5 & -15 \\ -15 & 55 \end{pmatrix} \begin{pmatrix} 68 \\ 19 \end{pmatrix} = \begin{pmatrix} 1.1 \\ 0.5 \end{pmatrix}$$

$$y = 1.1x + 0.5$$

$$E = [1 - f(1)]^2 + [3 - f(2)]^2 + [4 - f(3)]^2 + [6 - f(4)]^2 + [5 - f(5)]^2$$
$$= [1 - 1.6]^2 + [3 - 2.7]^2 + [4 - 3.8]^2 + [6 - 4.9]^2 + [5 - 6]^2 = 2.7.$$